Sources of errors

- Formulation errors
- Inherent errors
- Truncation errors
- Rounding errors
- Chopping errors
- Accumulated errors

The numerical solution for any problem is approximate value to the exact value

Numerical solution = exact solution + error

There are two methods for measuring errors

- **Absolute error** Let represents the approximate value to the exact value p , then the absolute error is defined as =
- **Relative error** = . p 0

Example If p = 0.300010 and = 0.310010 then = 0.1 and = 0.33

Accumation in error to estimate errors in the four operations

- **Addition**

(x+y) = (x+y)- () = (x-)+(y-) = x+ y

= = + x+ y

- **Subtraction**

(x-y) = (x-y)- () = (x-)-(y-) = x- y

= = = - = x- y

3)Multiplication

(xy) = (xy)- () =(xy)-)x - x)(y-y= (xy – xy+xy+ y x +xy= xy+ y x +xy = xy+ y x

= = = + = + x

4)Division

() = - = - = - = –( )() = –( )(1+) = –(+ y - -) = - - = - = (-)

= = = = - y

Floating point formula

Let x be a real number .There are two formulas for numbers in the floating point. In the first formula the number is written as x = A

Example x = 25149 = 0.25149

Y = - 0.0125 = - 0.125

z = -78.439 = - 0.78439

k = 0.733 = 0. 733

Let x = , y = . To addition or subtraction x and y must the conditions satisfies = ,

= . To multiplication or division x and y must the condition = .

Example If x = 22.159 , y = 0.03 and z = 111

Find 1) k =2x + 2) k = – yz 3) k = x – 2y + xz

x = 22.159 = 0.22159

y = 0.03 = 0.3

z = 111 = 0.111

2x = 2(0.22159) = 0.44318

y+z = 0.3 + 0.111 = 0.00003 + 0.111 = 0.11103

= = 0.501060517

2x + = 0.44318 + 0.501060517 = 0.44318 + 0.0501060517 = 0.4932860517

In the second formula the number is written as x =   +

Example write the following numbers in the floating point formula and three decimal places

x = 63249 = 0.63249 = 0.632 + 0.49

Y = - 1579.26 = - 0.157926 = - 0.157 +(- 0.926)

z = 0.01295 = 0.1295 = 0.129 + 0.5

k = 173.18 = 0.17318 = 0.173 + 0.18

Finding the absolute and relative errors

1) Errors in chopping case

Let x =   +   , =

(x) = = = =

(x) = =

(x) =

(x) =   =

2) Errors in chopping case

Let x = + , =

+

(x) = = = =

(x) = = =

(x) = = = = = 0.5

(x) = = =

Fundamental theorem of algebra

Every polynomial of degree n has n roots in complex numbers

Let f be a continuous function on [a , b]

If f(a) f(b) 0 then there exists roots in [a , b]

If f(a) f(b) 0 then there is no roots in [a , b]

Example Find the position of roots of f(x) = - 7+3+26x-10 in [-8,8]

x	-8	-6	-4	-2	0	2	4	6	8
F(x)	+	+	+	+	-	+	-	+	+

f(-2) f(0) 0 then there exists roots in [-2 , 0]

f(0) f(2) 0 then there exists roots in [0 , 2] f(2) f(4) 0 then there exists roots in [2 , 4]

f(4) f(6) 0 then there exists roots in [4 , 6]

1) Bisection methods

Example Find the root of x = in [0,1]

F(x) = x -

F(0) = -1 , f(1) = 0.632

f(0) f(1) 0 then there exists roots in [0 , 1]

= (0+1)/2 = 0.5 , f(0.5) = -0.1065

f(0) f(0.5) 0 then there is no roots in [0 , 0.5]

f(0.5) f(1) 0 then there exists roots in [0.5 , 1]

= (0.5+1)/2 = 0.75 , f(0.75) = 0.2776

f(0.75) f(1) 0 then there is no roots in [0.75 , 1]

f(0.5) f(0.75) 0 then there exists roots in [0.5 , 0.75]

= (0.5+0.75)/2 = 0.625 , f(0.625) = 0.0897

f(0.625) f(0.75) 0 then there is no roots in [0.625 , 0.75]

f(0.5) f(0.625) 0 then there exists roots in [0.5 , 0.625

Example Find the root of f(x) = - 7+3+26x-10 in [0,2]

F(0) = -10 , f(2) = 14

f(0) f(1) 0 then there exists roots in [0 , 2]

 = (0+2)/2 = 1 , f(1) = 13

f(1) f(2) 0 then there is no roots in [1 , 2]

f(0) f(1) 0 then there exists roots in [0 , 1]

 = (0+1)/2 = 0.5 , f(0.5) = 2.9375

f(0.5) f(1) 0 then there is no roots in [0.5 , 1]

f(0) f(0. 5) 0 then there exists roots in [0 , 0. 5]

 = (0. +0. 5)/2 = 0. 25 , f(0. 25) = -3.41796875

f(0) f(0.25) 0 then there is no roots in [0 , 0.25]

f(0.25) f(0. 5) 0 then there exists roots in [0.25 , 0. 5]

Stopping condition

1)

2)

3)

Example Find the root of f(x) = in [-1,1]

F(-1) = -1 , f(1) = 1

f(-1) f(1) 0 then there exists roots in [-1 , 1]

 = (-1+1)/2 = 0 , f(0) = 0 , the root is

Example Find the approximate positive value of the root of f(x) = cos (x) = x in 0,1]

with = 0.01

F(0) = 1 , f(1) = -0.46

f(0) f(1) 0 then there exists roots in [0 , 1]

 = (0+1)/2 = 0.5 , f(0.5) = 0.628

f(0) f(0.5) 0 then there is no roots in [0 , 0.5]

f(0.5) f(1) 0 then there exists roots in [0.5 , 1]

 = (0.5+1)/2 = 0.75 , f(0.75) = 0.169

 = 0.25   0.01

f(0. 5) f(0.75) 0 then there is no roots in [0. 5 , 0.75]

f(0.75) f(1) 0 then there exists roots in [0.75 , 1]

 = (0.75+1)/2 = 0.875 , f(0.875) = -0.125

 = 0.125   0.01

f(0.875) f(1) 0 then there is no roots in [0.875 , 1]

f(0.75) f(0.875) 0 then there exists roots in [0.75 , 0.875]

 = (075+0.875)/2 = 0.813 , f(0.813) = 0.026

 = 0.062   0.01

f(075) f(0.813) 0 then there is no roots in [0.75 , 0.813]

f(0.813) f(0.875) 0 then there exists roots in [0.813 , 0.875]

 = (0.813+0.875)/2 = 0.844 , f(0.844) = -0.048

 = 0.031   0.01

f(0. 844) f(0.875) 0 then there is no roots in [0. 844, 0.875]

f(0. 813) f(0.844) 0 then there exists roots in [0.813 , 0,844]

 = (0.813+0.844)/2 = 0.829 , f(0.829) = -0.011

 = 0.115   0.01

f(0.829) f(0.844) 0 then there is no roots in [0.829 , 0.844]

f(0.813) f(0.829) 0 then there exists roots in [0.813 , 0.829]

 = (0.813+0.829)/2 = 0.821 , f(0.821) = 0.007

 = 0.008   0.01

The root is

2) False position (Regular False method)

= - , i = 1 , 2 , 3 , ….

Example Find the root of f(x) = xlogx -1 in [1,2] with = 0.001

= 1 , = f( ) = -1 , = 2 , = f( ) = 0.3863

= - = 1.72134459 , = f( ) = -0.59402

= - = 1.89017687 , = f( ) = -0.477361475

= 0.16883228 0.001

= - = 1.76315 , = f( ) = -0.565755088

= 0.12702687 0.001

= - = 1.76322

= 0.00013 0.001

The root is

3)Secant method

= - , i = 0.1 , 2 , 3 , ….

Example Find the root of f(x) = x -1 in [-1,2] , = 0 , = 1 with = 0.05

= f( ) = -1, = f( ) = 1,718

= - = 0.368 , = f( ) = -0.468

= 0.4680.05

= - = 0.503 , = f( ) = -0.168

= 0.1680.05

= - = 0.580 , = f( ) = 0.036

= 0.0360.05

The root is

4) Newton Raphson method

Let f(x) be differentiable function on [a,b]

= - , n = 0 , 1 , 2 , 3 , ….

Example Find the root of f(x) = -1 with = 0

(x) = 2x + 2.1

= - = 0 4032

= - = 0.4

= - = 0.4

The root is

Example Find the square root of a number n

X = , = n

F(x) = – n , (x) = 2x

 = - = = - =)

Example Find

X =   , n = , F(x) = n - , (x) =

 = - = = - = (2-n)

5) Fixed point iterative theorem

A fixed point of a function g(x) is a real number x such that g(x) = x

The iteration = g() , n = 0 , 1 , 2 , ….is called fixed point iteration

Example Find the root of f(x) = - x -3 in [2,3] , = 2.5

X = 1 + = (x)

X = -3 = (x)

X = = (x)

X = = (x)

= ()	= ()	= ()	= ()
2.5	2.5	2.5	2.5
2.2	3.25	2.40625	2.3125
2.36364	7.5625	2.35828	2.302802
2.26923	54.1914	2.33288	2.302776
2.32203	2933.71	2.31920	2.302776
2.29197	8606642.63	2.31176	2.302776
2.30892	741	2.30770	2.302776

The function , , converges to the solution while diverges

The function g(x) is converges to the solution if 1

Example g(x) = 1 + x - , = -2.05

) = 1- , 1

= -2.05

= -2.100625

= -2.0378135

= -2.41794441

The sequence does not converge to x = -2

g(x) = 1 + x - , = 1.6

1

= 1.6

= 1.96

= 1.9996

= 1.99999996

The sequence converge to x = -2

Aitken formula for accelerating convergence

= - , n = 0 , 1 , 2 , 3 , ….

Example Find the root of f(x) = - x -3 in [2,3] , = 2.5

g(x) =

= 2.5

= g() = 2.40625

= g() = 2.35828

= - = 2.3080157

= g() = 2.3288

= - = 2.3042979

Consider the following system

(x,y) = 0 , (x,y) = 0

1) Fixed point iterative theorem

= ()

= () , , n = 0 , 1 , 2 , ….

The condition for converges is

L = max{+ , 1

Stopping condition and

Example

(x,y) = + -4 ,(x,y) = y - , = (1,1)

X = = (x,y)

Y = = (x,y)

L = max{+ , } = max{+ , }

L = max{+ , }= max{ , } = 1

= () = 1

= () =

= () =

= () =